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Radiative Corrections to the Inflaton Potential as an Explanation of Suppressed Large Scale Power in Density Perturbations and the Cosmic Microwave Background

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Abstract

The Wilkinson Microwave Anisotropy Probe microwave background data suggest that the primordial spectrum of scalar curvature fluctuations is suppressed at small wavenumbers. We propose a UV/IR mixing effect in small-field inflationary models that can explain the observable deviation in WMAP data from the concordance model. Specifically, in inflationary models where the inflaton couples to an asymptotically free gauge theory, the radiative corrections to the effective inflaton potential can be anomalously large. This occurs for small values of the inflaton field which are of the order of the gauge theory strong coupling scale. Radiative corrections cause the inflaton potential to blow up at small values of the inflaton field. As a result, these corrections can violate the slow-roll condition at the initial stage of the inflation and suppress the production of scalar density perturbations.

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1 Introduction

High precision observational cosmology places tight constraints on the cosmological parameters of our universe. The Wilkinson Microwave Anisotropy Probe (WMAP) results measuring anisotropies in the Cosmic Background Radiation (CBR) [1, 2], when combined with data on high redshift supernovae and large scale structure, strongly support the "concordance model" for our universe[1]. One of the intriguing observations of the CBR anisotropy spectrum is the suppression of its low-ℓ multipoles compared to the predictions of ΛCDM model. The low-ℓ multipoles of the temperature-temperature (TT) angular power spectrum correspond to large angular scales — they encode the information about the small wavenumbers in the spectrum of primordial density perturbations. Consequently they provide a window on the detailed features of the inflaton potential during the part of inflation [3] that gives rise to observables in structure formation and the microwave background, produced roughly 60-50 e-foldings before the end of inflation. Unfortunately, it is not possible to disentangle the suppression of low-ℓ modes from cosmic variance limitations. In this paper we assume the suppression is a physical effect.

Not surprisingly, this suppression of low-l multipoles (along with the possibility of a running spectral index) was the focus of much recent discussion [4–20]. Most previous work attempted to identify a new ultraviolet (UV) physics responsible for the suppression of the primordial power spectrum at small wavenumbers. In this paper we point out a simple low-energy (albeit strongly coupled) field-theoretic phenomenon that produces a 'feature' in the inflaton potential that serves to explain the suppression of low-ℓ CBR anisotropies[2]. The observed effect is specific to small-field inflationary models [22, 23]. Here, small-field models are defined to be those models of inflation in which the initial value of the scalar field is small (e.g. near zero) as it starts rolling down the potential.

We find a phenomenon that modifies the usual power spectrum of density fluctuations from inflation,

$$\left(\delta_H^{(0)}\right)^2 \propto \left(\frac{k}{k_s}\right)^{n_s-1}, \tag{1.1}$$

[1]The concordance model is a spatially flat Universe with an adiabatic, nearly scale invariant spectrum of initial fluctuations. In what follows we assume a ΛCDM model as a realization of the concordance model.

[2]Effects of strongly coupled gauge theory dynamics on inflation were also studied in [21].

to

$$\delta_H^2 = \left(\delta_H^{(0)}\right)^2 \ f\left(\frac{k}{k_s}\right) , \tag{1.2}$$

where the superscript 0 refers to the unmodified spectrum, and the form-factor $f(\phi)$ has the following behavior:

$$f(k) \to 0 \ \ \text{(small } k\text{)} \tag{1.3}$$

$$f(k) \to 1 \ \ \text{(large } k\text{).} \tag{1.4}$$

Here k_s corresponds to the scale of perturbations that are produced 60 e-foldings before the end of inflation, which corresponds to the present horizon size, $k_s \sim (4000Mpc)^{-1}$.

The suppression of large-scale power arises because the potential blows up (becomes infinite) at small values of the field, so that the field is not slowly rolling at all and production of density fluctuations is suppressed; see Fig. (1). This blow-up of the potential is sharply localized at 60 e-foldings before the end of inflation, so that ordinary density fluctuations and structure formation ensue just afterwards (at 60-50 e-foldings before the end). While the sharpness of the feature is generic to our mechanism, its location at exactly 60 e-folds before the end requires fine tuning[3]. The origin of the blow-up of the potential is due to the radiative corrections for an inflaton coupled to an asymptotically free gauge theory (analogous to QCD).

Thirty years ago Coleman and Weinberg computed the one-loop effective potential for (classically) conformally invariant SU(2)×U(1) gauge theory with no quadratic mass term. However, due to the massive top quark, the contributions due to the t-quark's Yukawa coupling constant have subsequently been found to dominate over those of the SU(2) × U(1) gauge coupling constants. Moreover, when there is a large Yukawa coupling, subsequent leading logarithm terms to the one-loop effective potential are too large to neglect. In the next section we review a technique developed in Ref. [25,26] for all-order summation of leading logarithm terms for the effective potential, including coupling to asymptotically free theory (QCD). Radiative symmetry breaking as an explanation for the Higgs mass is revived in this context, since a prediction for the Higgs mass above 200 GeV can now be accommodated [25].

The physics characterizing this model extends to more general scalar potentials, including that of any inflaton coupled to an asymptotically free gauge theory. We

[3]The same fine-tuning problem is acknowledged in all previous work on the small-ℓ suppression of CBR power spectrum we are aware of [4–20]. Unfortunately, neither of the proposed models provided the solution to it. It would be very interesting to identify physical phenomena that alleviates the latter fine tuning problem.

explicitly compute the leading contribution to the effective inflaton (Higgs) potential and show that radiative corrections become anomalously large for small vacuum expectation values of the inflaton. This is a reflection of a new UV/IR mixing effect in the model: even though the inflationary potential has a typical GUT scale height, the effective QCD coupling in radiative corrections to the inflaton potential is evaluated at the field value of the inflaton. The effect is thus most profound if the inflaton value at the beginning of inflation is close to the gauge theory's strong coupling scale. Thus, for a sufficiently small initial inflaton value, the QCD coupling develops a perturbative Landau pole that strongly enhances radiative corrections.

In section 3 we model these radiative corrections in small-field inflationary scenarios and demonstrate that they suppress the primordial power spectrum of scalar density fluctuations for small wavenumbers. We use the CMBEASY package [27] to relate the latter suppression to the small multipole suppression in the CBR spectrum of anisotropies as observed by WMAP. We summarize our results in section 4.

2 QCD contributions to radiative Higgs potential

A technique for all-order summation of leading logarithm terms for the effective potential in radiative electroweak symmetry breaking, including coupling to asymptotically free theory (QCD), has been developed in Ref. [25, 26]. In this section we review this analysis.

Though our discussion is in the context of radiative symmetry breaking of the Higgs potential in the Standard Model, the physics of the ultraviolet-infrared mixing characterizing this model extends to more general scalar (inflaton) potentials: all that is required is the coupling of the inflaton to an asymptotically free gauge theory. We emphasize the latter genericity, as from the inflationary perspective the radiative symmetry breaking potential discussed in this section is not suitable for inflation: it predicts unacceptably large amplitude of density fluctuations. In the case of the effective radiative symmetry breaking Higgs potential in the Standard Model the dominant QCD contribution is given by (2.19), which is the main result of the section. We expect perturbatively divergent contributions of precisely this kind to be present in all small-field inflationary models with inflaton coupling to a QCD-like gauge theory. Hence, in the next section we consider an exponential potential for inflation (different from what is considered in this section), but with the same physical behavior due to coupling to

an asymptotically free theory.

In the absence of an explicit scalar-mass term, the one-loop effective potential for SU(2) $\times$ U(1) gauge theory was computed in 1973 in the seminal paper by Coleman and Weinberg [28]

$$V_{eff}^{1-loop} = \frac{\lambda\phi^4}{4} + \phi^4 \left[\frac{3\lambda^2}{16\pi^2} + \frac{3(3g_2^4 + 2g_2^2 g'^2 + g'^4)}{1024\pi^2}\right] \left(\log\frac{\phi^2}{\mu^2} - \frac{25}{6}\right) \tag{2.1}$$

where the $-25/6$ constant is chosen to ensure that

$$\frac{d^4 V_{1-loop}}{d\phi^4} = \frac{d^4 V_{tree}}{d\phi^4} = 6\lambda \tag{2.2}$$

and $\{g_2, g'\}$ are the SU(2) and U(1) coupling constants respectively.

The above one-loop computation neglects quark Yukawa couplings. As there is a heavy t-quark, this is no longer justifiable[4]. In fact [26], the Yukawa coupling of the t-quark makes contributions to the scalar effective potential V_{eff} which *dominate* over those of the SU(2) $\times$ U(1) gauge coupling constants. As in [26], we neglect the SU(2) $\times$ U(1) gauge couplings, and all quark Yukawa couplings except for that of the t-quark. Thus, the effective potential takes the form

$$V_{eff} = V_{eff}\left(\lambda(\mu), g_t(\mu), g_3(\mu), \phi^2(\mu), \mu\right) , \tag{2.3}$$

where λ is the quartic scalar-field self-interaction coupling constant appearing in the tree-level scalar potential

$$V_{tree} = \frac{\lambda}{4}\phi^4 + \text{const} \equiv \frac{\lambda}{4}\phi^4 + V^{(0)} , \tag{2.4}$$

g_t is the Yukawa coupling of the t-quark, g_3 is the QCD coupling, and μ is the renormalization mass scale. The requirement that V_{eff} is independent of the renormalization scale μ gives rise to the familiar renormalization group equation for the effective potential

$$\begin{aligned} 0 =& \mu\frac{d}{d\mu} V[\lambda(\mu), g_t(\mu), g_3(\mu), \phi^2(\mu), \mu] \\ =& \left(\mu\frac{\partial}{\partial\mu} + \beta_\lambda\frac{\partial}{\partial\lambda} + \beta_t\frac{\partial}{\partial g_t} + \beta_3\frac{\partial}{\partial g_3} - 2\gamma\phi^2\frac{\partial}{\partial\phi^2}\right) V\left(\lambda, g_t, g_3, \phi^2, \mu\right) , \end{aligned} \tag{2.5}$$

[4] When there is a large Yukawa coupling, it is also inconsistent to neglect higher-loop contributions to the effective potential.

where to one loop order in λ, g_t and g_3

$$\begin{aligned}
\beta_\lambda &\equiv \mu\frac{d\lambda}{d\mu} = \frac{48\lambda g_t^2}{64\pi^2} + \frac{12\lambda^2}{8\pi^2} - \frac{3g_t^4}{8\pi^2} + \mathcal{O}\left(\lambda^k g_{3,t}^{6-2k}\right) \ , \\
\beta_t &\equiv \mu\frac{dg_t}{d\mu} = \frac{\frac{9}{2}g_t^3 - 8g_t g_3^2}{16\pi^2} + \mathcal{O}\left(\lambda^k g_{3,t}^{5-2k}\right) \ , \\
\beta_3 &\equiv \mu\frac{dg_3}{d\mu} = -\frac{7g_3^3}{16\pi^2} + \mathcal{O}\left(g_{3,t}^5\right) \ , \\
\gamma &\equiv -\frac{\mu}{\phi}\frac{d\phi}{d\mu} = \frac{3g_t^2}{16\pi^2} + \mathcal{O}\left(\lambda^k g_{3,t}^{4-2k}\right) \ .
\end{aligned} \tag{2.6}$$

For small coupling constants $\{x, y, z\}$ defined at a scale $\mu = 2^{-1/4}G_F^{-1/2} \equiv v$

$$\begin{aligned}
x &\equiv g_t^2(v)/4\pi^2 \, , \\
y &\equiv \lambda/4\pi^2 \, , \\
z &\equiv g_3^2(v)/4\pi^2 \, ,
\end{aligned} \tag{2.7}$$

the summation-of-leading-logarithms effective Higgs potential can be written as [26]

$$V_{eff}^{LL} \equiv \pi^2\phi^2 S_{LL} = \pi^2\phi^4\left\{\sum_{n=0}^{\infty} x^n \sum_{k=0}^{\infty} y^k z^l C_{n,k,l} L^{n+k+l-1}\right\} \ , \ (C_{0,0,0} = 0) \, , \tag{2.8}$$

where the series S_{LL} is the sum of all contributions involving a power of the logarithm

$$L \equiv \ln(\phi^2/\mu^2) \tag{2.9}$$

that is only one degree lower than the aggregate power of the couplings $\{x, y, z\}$. In this approximation the RGE (2.5) takes the following form

$$\left[-2\frac{\partial}{\partial L} + \left(\frac{9}{4}x^2 - 4xz\right)\frac{\partial}{\partial x} + \left(6y^2 + 3yx - \frac{3}{2}x^2\right)\frac{\partial}{\partial y} - \frac{7}{2}z^2\frac{\partial}{\partial z} - 3x\right] S_{LL}(x, y, z, L) = 0 \, . \tag{2.10}$$

Remarkably a closed-form solution to (2.10) can be written down [26]

$$V_{eff}^{LL} = \pi^2\bar{y}(L/2)\bar{\phi}^4(L/2) = \pi^2\bar{y}(L/2)\phi^4\exp\left[-3\int_0^{L/2}\bar{x}(t)dt\right] \ , \tag{2.11}$$

where $\{\bar{x}(t), \bar{y}(t), \bar{z}(t)\}$ are characteristic functions defined by the differential equations and initial conditions

$$\frac{d\bar{z}}{dt} = -\frac{7}{2}\bar{z}^2 \, , \qquad \bar{z}(0) = z \, , \tag{2.12}$$

$$\frac{d\bar{x}}{dt} = \frac{9}{4}\bar{x}^2 - 4\bar{x}\bar{z}\,, \qquad \bar{x}(0) = x\,, \tag{2.13}$$

$$\frac{d\bar{y}}{dt} = 6\bar{y}^2 + 3\bar{x}\bar{y} - \frac{3}{2}\bar{x}^2\,, \qquad \bar{y}(0) = y\,, \tag{2.14}$$

$$\frac{d\bar{\phi}}{dt} = -\frac{3}{4}\bar{x}\bar{\phi}\,, \qquad \bar{\phi}(0) = \phi\,, \tag{2.15}$$

Eq.(2.12) describes the running of the QCD coupling

$$\bar{z}(t) = \frac{2z}{2 + 7z\ t}\,. \tag{2.16}$$

For

$$t \equiv t_s = -\frac{2}{7z}\,, \tag{2.17}$$

the QCD coupling blows-up — this is the standard IR Landau pole of the asymptotically free gauge theories. It is straightforward to evaluate V_{eff}^{LL} in the vicinity of the pole, *i.e.*, $|L/2 - t_s| \to 0$. Given that

$$\begin{aligned} \bar{x}(t) &= \frac{4z}{9(2+7zt)}\left[1 + \cdots\right]\,, \\ \bar{y}(t) &= \frac{\xi z}{2+7zt}\left[1 + \cdots\right]\,, \qquad \xi \equiv \frac{\sqrt{689} - 25}{36}\,, \end{aligned} \tag{2.18}$$

where $\cdots$ indicates terms vanishing in the limit $(t - t_s) \to 0$, we find

$$V_{eff}^{LL} = \frac{\mu^4\pi^2\xi z}{2}\ e^{-\frac{8}{7z}}\ \left(\frac{9x}{2z}\right)^{4/3}\ \left(1 + \frac{7}{4}z\ L\right)^{-25/21}\left[1 + \mathcal{O}\left(1 + \frac{7}{4}zL\right)\right] \tag{2.19}$$

as $(L/2 - t_s) \to 0$, i.e., in the vicinity of the pole. Notice that the dominant contribution near the perturbative QCD pole in (2.18) is insensitive to the small UV values of the top quark Yukawa coupling (x) and Higgs self-coupling (y), (2.12)-(2.14). On the other hand, the residue of the branch-cut in the effective potential (2.19) does depend on the UV top quark Yukawa coupling[5] as this singularity arises *only* when this coupling is non-vanishing. Leading singular behavior of (2.18), (2.19) reflects the fact that an exact solution of (2.12)-(2.14)

$$\begin{aligned} \bar{z}(t) =& \frac{2z}{2 + 7z\ t}\,, \\ \bar{x}(t) =& \frac{4z}{9(2+7zt)}\,, \\ \bar{y}(t) =& \frac{\xi z}{2+7zt}\,, \end{aligned} \tag{2.20}$$

[5]Such dependence arises from the subdominant terms in $\bar{x}$, (2.18).

with initial conditions

$$\bar{z}(0) = z, \qquad \bar{x}(0) = \frac{2z}{9}, \qquad \bar{y}(0) = \frac{\xi z}{2}, \tag{2.21}$$

is an attractor of the physical[6] renormalization group flow. Finally, the radiative contribution dominance over the tree level Higgs potential in the full effective potential (2.19) is sharply localized near the QCD strong coupling scale.

The *existence* of this perturbative singularity is generic for all asymptotically free gauge theories coupled to an inflaton (Higgs) field (n.b. the order of the branch-cut and its residue in the leading-logarithms effective potential (2.19) is specific to the Standard Model matter content). The point is simply that radiative corrections to the tree-level classical inflaton potential become anomalously large when evaluated for small values of the inflaton field, a consequence of the perturbative IR pole of the asymptotically free gauge theory. In the framework of the effective quantum field theory, the above conclusion is independent of the scale of the tree-level inflaton potential, the constant term in (2.4). In fact, both $V^{(0)}$ and v (see (2.7)) can be of order the GUT scale, and V_{eff} would still have a perturbative singularity for sufficiently small ϕ. Hence in the next section we will turn to inflation.

3 UV/IR mixing in small-field inflationary models

In the previous section we argued that radiative corrections to the tree-level effective inflaton potential are important, provided the value of the inflaton is of the same order as the strong coupling scale of the asymptotically free gauge theory to which it couples. In what follows we study a simple model of small-field inflation that illustrates observed effects on the spectrum of CBR anisotropies. Here, small-field models are defined to be those models of inflation in which the initial value of the scalar field is small (e.g. near zero) as it starts rolling down the potential. Examples of small-field models include models based on spontaneous symmetry breaking phase transitions where the field rolls away from an unstable equilibrium such as natural inflation [24]. In this paper, as a simple example, we will consider an exponential potential.

We work with the tree level inflaton potential [29–31]

$$V_{tree} = V_0 e^{-\lambda\phi}, \tag{3.1}$$

[6] Subject to the constraints $\{\bar{x}(t), \bar{y}(t), \bar{z}(t)\} > 0$.

where $\phi \equiv \Phi/m_{pl}$ is the inflaton field in Planck units. We assume that the inflaton couples to some asymptotically free gauge GUT. As a result, the tree-level inflaton potential will receive radiative corrections. We model radiative corrections to (3.1) as

$$V_{eff} = V_{tree} + V_{radiative} \equiv V_0 e^{-\lambda\phi} + \alpha \, \frac{V_0}{\ln \frac{\phi}{\Lambda}} \,, \tag{3.2}$$

where α is proportional to the coupling constant of the GUT asymptotically free gauge theory at the GUT scale, thus $0 < \alpha \ll 1$. The quantity Λ is the strong coupling scale of the gauge theory in Planck units, $\Lambda \ll 1$. The form of the potential is similar to that of Fig. (1); the potential becomes infinite at $\phi \to \Lambda$. The starting point for inflation then clearly has to be at $\phi = \phi_i > \Lambda$.

The effective potential V_{eff} has the required features to explain the suppression of the observed CBR anisotropy spectrum at small l. Indeed, if the beginning of inflation[7] ϕ_i is close to Λ, *i.e.*, $(\phi_i - \Lambda) \ll \Lambda$, the radiative contribution in (3.2) is dominant. The inflaton then rolls too fast to effectively generate primordial density perturbations at small wavenumbers. This will further translate into the suppression of the CBR anisotropy spectrum at large angles (or small l). This suppression is rather sharply localized: for $\phi \sim 2\Lambda$ or larger, the radiative contribution does not exceed V_{tree}:

$$V_{radiative} \propto \alpha \, V_0 \ll V_0 \approx V_{tree} \,. \tag{3.3}$$

We now turn to the quantitative analysis of inflation with (3.2). First, to determine the scales and parameters we set $\alpha = 0$, thus $V_{eff} = V_{tree}$. The standard slow roll parameters are[8]

$$\begin{aligned} \epsilon &= \frac{m_{pl}^2}{2} \left(\frac{V'_{eff}}{V_{eff}} \right)^2 = \frac{1}{2} \lambda^2 \,, \\ \eta &= m_{pl}^2 \left(\frac{V''_{eff}}{V_{eff}} \right) = \lambda^2 \,, \end{aligned} \tag{3.4}$$

producing scalar density fluctuations with power spectrum

$$\delta_H^2 \bigg|_{\alpha=0} \equiv \left(\delta_H^{(0)} \right)^2 \propto \left(\frac{k}{k_s} \right)^{n_s - 1} \,, \tag{3.5}$$

with spectral index

$$n_s - 1 = -6\epsilon + 2\eta = -\lambda^2 \,. \tag{3.6}$$

[7] By the 'beginning' we mean the very outset of the ~ 60 e-foldings characterizing inflation.

[8] Prime denotes derivative with respect to Φ.

In (3.5) k_s corresponds to the scale of perturbations that are produced 60 e-foldings before the end of inflation, which corresponds to the present horizon size, $k_s \sim (4000Mpc)^{-1}$. Within the slow-roll approximation, adiabatic density perturbations are given by

$$\delta_H^{(0)} \sim \frac{1}{\sqrt{75}\pi m_{pl}^3} \frac{V_{eff}^{3/2}}{V'_{eff}} = \frac{V_0^{1/2}}{\sqrt{75}\pi m_{pl}^2 \lambda} \, e^{-\lambda\phi/2} \, . \tag{3.7}$$

This quantity should equal $1.91 \cdot 10^{-5}$ at about $N_e \sim 60$ e-foldings before the end of inflation. Assuming $\lambda\phi_i \ll 1$ and $m_{pl} = 2.4 \cdot 10^{18}$ GeV, for $\lambda \sim 0.1$ we find that $V_0^{1/4} \sim 10^{16}$ GeV from (3.7). If inflation ends at $\phi = \phi_f$, the number of e-foldings N_e is

$$N_e = -\frac{1}{m_{pl}^2} \int_i^f \frac{V_{eff}}{V'_{eff}} \, d\Phi = \frac{1}{\lambda} \int_i^f d\phi = \frac{\triangle\phi}{\lambda} \, . \tag{3.8}$$

Thus to get sufficient inflation, $\triangle\phi \sim 10$. Since derivatives with respect to ϕ can be expressed with respect to wavenumber k as

$$\frac{m_{pl}}{2} \sqrt{\epsilon} \frac{d}{d\Phi} = (1-\epsilon) \frac{d}{d \ln k} \, , \tag{3.9}$$

or

$$\frac{\lambda}{2^{3/2}} \frac{d}{d\phi} \approx \frac{d}{d \ln k} \, , \tag{3.10}$$

we can relate the expectation value of the inflaton and the scale of perturbations k leaving the horizon at the corresponding instant during inflation

$$\phi(k) \approx \phi_i + \frac{\lambda}{2^{3/2}} \ln \frac{k}{k_s} \, . \tag{3.11}$$

Upon inclusion of the radiative correction to the tree-level potential (3.1), the primordial spectrum of density perturbations (3.5) will be modified according to

$$\delta_H^2 = \left(\delta_H^{(0)}\right)^2 \, f\left(\frac{k}{k_s}\right) \, , \tag{3.12}$$

where

$$\left(\delta_H^{(0)}\right)^2 \sim \frac{1}{\sqrt{75}\pi m_{pl}^3} \frac{V_{tree}^3}{V'^2_{tree}} \tag{3.13}$$

and the form-factor $f(\phi)$ can be estimated using the slow-roll approximation in Eq.(3.7) and Eq.(3.2),

$$f \approx \frac{\left(1 + \frac{V_{radiative}}{V_{tree}}\right)^3}{\left(1 + \frac{V'_{radiative}}{V'_{tree}}\right)^2} = \frac{\left(1 + \frac{1}{\ln \frac{\phi}{\Lambda}} \, \alpha e^{\lambda\phi}\right)^3}{\left(1 + \frac{1}{\frac{\phi}{\Lambda} \ln^2 \frac{\phi}{\Lambda}} \, \frac{\alpha e^{\lambda\phi}}{\lambda\Lambda}\right)^2} \, . \tag{3.14}$$

For inflaton expectation values ϕ close to the gauge theory strong coupling scale Λ (where $V_{radiative} >> V_{tree}$), we find that

$$f(\phi) \sim \alpha\lambda^2\Lambda^2 \ \ln\frac{\phi}{\Lambda}\,, \qquad (\phi-\Lambda) \ll \Lambda\,. \tag{3.15}$$

As $\phi/\Lambda \sim \mathcal{O}(1)$, $f(\phi)$ rapidly approaches zero. More precisely, assuming $\frac{\alpha}{\lambda\Lambda} > 1$, the transition between the small ϕ-regime (3.15) and $f(\phi) = 1 + \mathcal{O}(1)$ occurs within

$$\triangle\left(\frac{\phi}{\Lambda}\right) \sim \frac{\alpha}{\lambda\Lambda} \equiv \zeta \tag{3.16}$$

or, equivalently, using (3.11),

$$\triangle\left(\frac{k}{k_s}\right) \sim \delta \equiv \exp\left(\frac{2^{3/2}\zeta\Lambda}{\lambda}\right) = \exp\left(\frac{2^{3/2}\alpha}{\lambda^2}\right). \tag{3.17}$$

With $\phi_i \sim \Lambda$, and using (3.11), we can approximate

$$\begin{aligned}
f\left(\frac{k}{k_s}\right) &= 0, \qquad \frac{k}{k_s} < 1\,,\\
f\left(\frac{k}{k_s}\right) &= \frac{\ln\frac{k}{k_s}}{\ln\delta}, \qquad 1 \le \frac{k}{k_s} \le \delta\,,\\
f\left(\frac{k}{k_s}\right) &= 1, \qquad \frac{k}{k_s} > \delta\,,
\end{aligned} \tag{3.18}$$

Equivalently, using Eq.(3.11), the form-factor in the nontrivial region of (3.18) can be approximated as having a linear dependence on ϕ,

$$f(\phi) = \frac{\phi-\Lambda}{\Lambda\zeta}, \qquad 0 \le (\phi-\Lambda) \le \Lambda\zeta\,. \tag{3.19}$$

Given the modified primordial power spectrum (3.12) (with (3.5),(3.6) and the form-factor (3.18)), the present day power spectrum $P(k)$ is obtained as

$$\frac{k^3}{2\pi}P(k) = \left(\frac{k}{aH_0}\right)^4 \ T^2(k)\ \delta_H^2(k)\,, \tag{3.20}$$

where $T(k)$ is a transfer function as in [29], and H_0 is the present Hubble value.

In Fig. (2) we show the result generated with the CMBEASY program [27], which provides the power spectra conversion (3.20). In the latter analysis we take $\lambda = .2$ and adjust cosmological parameters as given by the best fit model of the WMAP collaboration. The two solid lines correspond to a choice of $\delta = \{2, 4\}$ in (3.18). The

dashed line corresponds to turning off the radiative correction, *i.e.*, setting $\alpha = 0$. For $\delta = 4$ we find from (3.18) $\alpha \approx 0.02$. Thus consistency of above approximations, *i.e.*, $\xi >> 1$ requires $\Lambda \ll 10^{-1}$. The latter is consistent with the assumption that the strong coupling scale of the GUT gauge theory is below the Planck scale.

We comment here on the validity of our approximation. Small-k suppression form-factor (3.18) has been obtained in the slow-roll approximation, which strictly speaking, is violated at the initial stages of inflation. Alternatively, one can argue that the initial stage of inflation can be described by "kinetic regime"[9], where the suppression form-factor was found (numerically) to be

$$f\left(\frac{k}{k_s}\right) \propto \left(\frac{k}{k_s}\right)^a \tag{3.21}$$

where $a \approx 3.35$ [9]. Though result (3.21) was obtained in the context of chaotic (large-field) inflation, it is straightforward to see that it is universal as long as inflaton energy is predominantly kinetic. Indeed, the primordial power spectrum is

$$\delta_H \simeq \left(\frac{\delta\rho}{\rho + p}\right)_{k=aH} \tag{3.22}$$

where in the kinetic regime (assuming $\delta\dot{\phi} = \frac{d}{dt}\delta\phi$)

$$\delta\rho \simeq \dot{\phi}\ \delta\dot{\phi} \simeq \dot{\phi}\ \frac{\dot{H}}{2\pi} \tag{3.23}$$

Using $\rho + p = -\dot{H}m_{pl}^2/(4\pi)$, we find

$$\delta_H \propto -\frac{\dot{\phi}}{m_{pl}^2}\bigg|_{k=aH} \tag{3.24}$$

where the cosmic time dependence in $\dot{\phi} = \frac{d}{dt}\phi(t)$ is assumed to be eliminated through the horizon crossing condition for the comoving momentum k as $k = a(t)H$. In the kinetic regime $\dot{\phi}^2 \gg V(\phi)$, thus expansion of the Universe is well approximated by a fluid with equation of state $p = \rho$. For such an expansion we have $\rho \simeq \frac{1}{2}\dot{\phi}^2 \propto a^{-6}$ and $a \propto t^{1/3}$, which in turn implies that $\dot{\phi} \propto -t^{-1}$. From the Friedmann equation we conclude that $H \propto \rho^{1/2} \propto t^{-1}$, and thus the comoving scale k crossing the horizon at cosmic time t is $k = a(t)H \propto t^{-2/3}$. Comparing the latter scale with the $\dot{\phi}$ time evolution we conclude

$$\dot{\phi} \propto -k^{3/2} \tag{3.25}$$

[9] This is inflationary stage where the dominant contribution to the inflaton energy comes from its kinetic energy.

leading to

$$\delta_H^2 \propto \left(\frac{k}{k_s}\right)^3 \tag{3.26}$$

which approximately reproduces numerical analysis of [9]. Even though the suppression form-factor in the primordial power spectrum (3.26) has power-law rather than logarithmic behavior (as in (3.18)), we find that the corresponding CBR spectrum of anisotropies is very similar to the one presented in Fig.2 (also compare with [13]).

4 Summary

WMAP data indicates the suppression of small-ℓ multipoles in the anisotropy power spectrum of the CBR. It is difficult to isolate this suppression from the cosmic variance limitations. However, when taken seriously, such suppression can be due to a suppression of power in the spectrum of primordial scalar density fluctuations during inflation. As the generation of quantum fluctuations during inflation is most efficient in the slow-roll regime, the CBR anisotropy measurements provide a signature for the breakdown of the slow-roll conditions during the first few e-foldings of the single field inflationary models. In this paper we have identified a low-energy field-theoretic phenomenon that arises generically in small-field inflationary models in which the inflaton is coupled to an asymptotically free GUT-like gauge theory. Here, the small vacuum expectation values of the inflaton field probe the strongly coupled dynamics of this gauge theory. This produces large radiative corrections to the tree-level inflaton potential which break its slow-roll conditions. The effect is most profound if the inflaton scale at the beginning of inflation is close to the gauge theory's strong coupling scale.

In this paper we have used summation of leading logarithms in the perturbation theory as a model of a strongly coupled dynamics of an inflaton and asymptotically free gauge theory. It would be very interesting to explore nonperturbative effects of the quantum field theory in this setting, particularly space-time curvature corrections.

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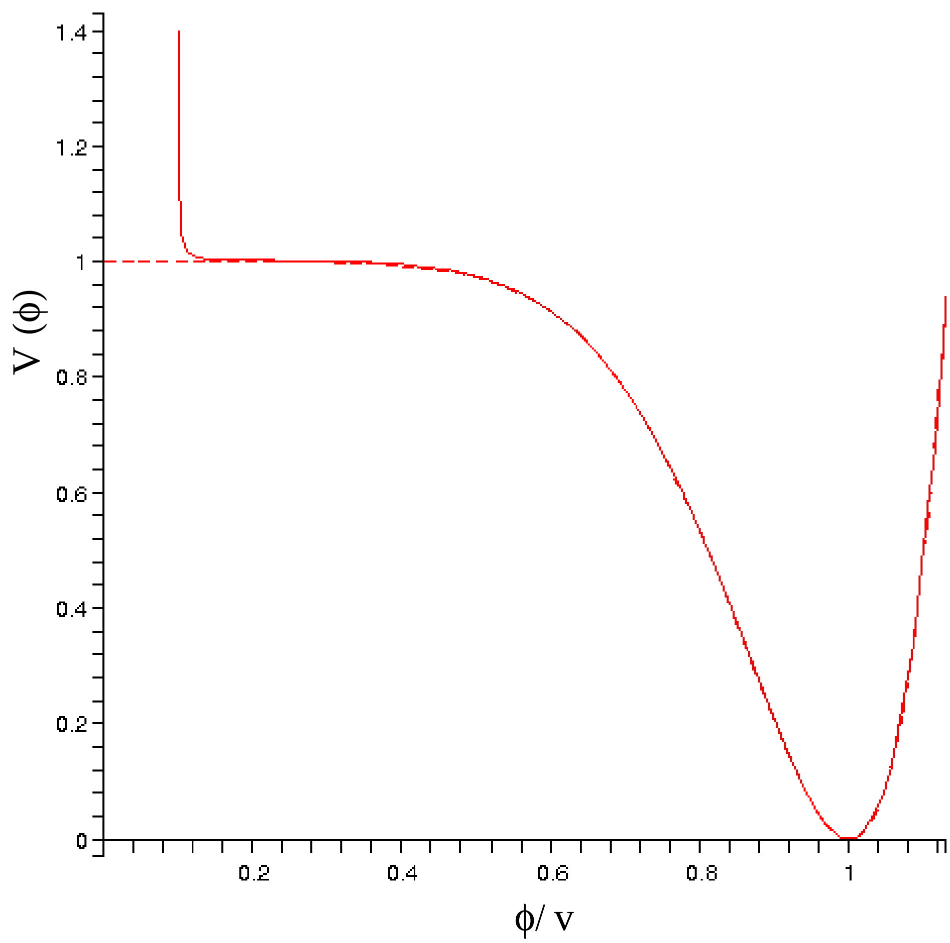

Figure 1: Effective inflaton potential (solid line) in small-field inflationary models with inflaton coupled to asymptotically free gauge theory. The slow-roll tree-level potential (dashed line) receives large radiative corrections whenever the inflaton expectation value is close to the gauge theory strong coupling scale $\phi \sim \Lambda \ll v$. In the sharply localized regime $|\phi - \Lambda| \ll v$ the field is not slowly rolling, and production of density fluctuations is strongly suppressed.

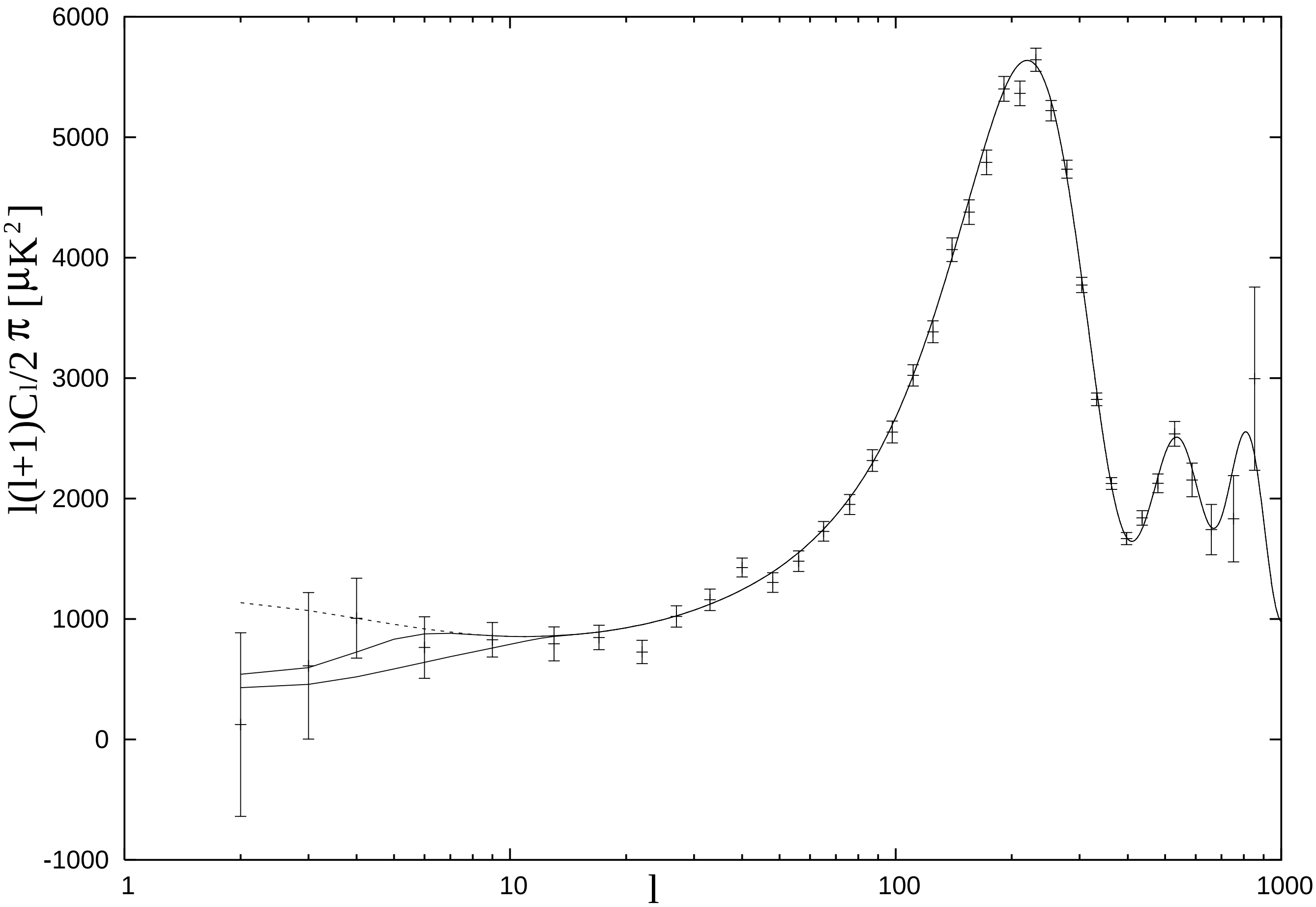

Figure 2: CBR spectrum of anisotropies with the primordial spectrum of scalar density fluctuations for the effective inflaton potential (2.3). We set $\lambda = 0.2$, $k_s = 0.00025\ Mpc^{-1}$. The scale of the tree level potential is $V_0^{1/4} \sim 10^{16}$ GeV. Two solid lines correspond to $\delta = \{2, 4\}$ in the form-factor (3.18), with larger δ corresponding to stronger suppression at small ℓ. The dashed line corresponds to the anisotropy spectrum from a tree-level approximation in the effective potential (2.3), *i.e.*, setting $\alpha = 0$. Cosmological parameters are as given by the best fit of the WMAP collaboration.